

A transparent box in the shape of a square prism has base edge lengths of 7 units and a height of 3 units. There are 35 identical cubes, all of which have a unit edge length. These cubes are stacked to be side-by-side and glued inside, starting from the base with 25 cubes in the first layer, then 9 cubes on them in the second layer, and finally one cube on top of them in the third layer. Then, a quantity of water is poured into the container and the lid is closed. When the base of this container is parallel to the ground, its height from the bottom of the water is 2 units. If the cube is inverted upside down so that the base remains parallel to the ground, what will be its height from the bottom of the water in units? A) $\frac{3}{2}$ B) $\frac{5}{3}$ C) $\frac{6}{5}$ D) $\frac{7}{5}$ E) $\frac{8}{3}$

Answer from Mathful

Thought for 228 seconds

I need to solve this problem. There's a transparent box that's a square prism with a base of 7 by 7 units and a height of 3 units. Inside, there are 35 unit cubes stacked: 25 on the bottom layer, 9 on the second, and one on the top. Water is poured in, and when the box is sitting normally, the water height from the bottom is 2 units. Then, we invert the box so it's upside down, but the base is still parallel to the ground, and we need to find the new water height.

First, I should figure out the volume of water and how it's distributed when the box is upright and after it's inverted.

Let me sketch this out. The box has a base area of $7 \times 7 = 49$ square units. Height is 3 units, so total volume is $49 * 3 = 147$ cubic units.

Now, there are cubes inside: 25 on bottom, 9 on second layer, one on top. Each cube is $1 \times 1 \times 1$, so volume occupied by cubes is 35 cubic units.

But the cubes are stacked, so they take up space, and water is around them.

When water is poured, with the lid closed, and the water height is 2 units from the bottom when upright.

But the cubes are there, so the water level is at 2 units, but since there are cubes, the actual water volume might be less because the cubes displace water or something.

I need to think about the space available for water.

When the box is upright, the base is 7×7 , and cubes are stacked starting from the bottom.

Bottom layer: 25 cubes, which is a 5×5 grid since $5^2 = 25$, but the base is 7×7 , so there's space around. 7×7 has 49 squares, 25 cubes, so 24 empty spaces on the bottom layer? But the cubes are stacked, so for the water, it's the volume between the cubes.

Perhaps I should think in terms of the cross-section or something.

Since the water height is given from the bottom, and it's 2 units, I need to see up to where the water fills, considering the cubes.

Let me think about the layers.

Bottom layer: 25 cubes, covering a 5×5 area in the 7×7 base. So, there are empty regions around the sides and also in the corners? 7×7 grid, 5×5 cubes in the center, I assume, since it says "stacked to be side-by-side and glued inside", but it doesn't specify the position, but probably centered or something? It doesn't matter for the area, I think, because the base area is fixed.

The base area for water to flow is the entire base minus the footprint of the cubes, but since cubes are there, the water can be in the gaps.

For the water height, when we say height from bottom is 2 units, that means the water surface is at 2 units above the bottom of the container.

Now, at different heights, the cross-sectional area available for water changes because of the cubes.

So, I need to find the volume of water by integrating the area or something, but since it's discrete layers, I can think per unit height.

The container height is 3 units, and cubes are in layers.

Let me define the height from the bottom.

From $z=0$ to $z=1$: this is the first unit height. In this layer, there are cubes present only where the bottom layer cubes are. The bottom layer has 25 cubes, each 1×1 , so in the area where cubes are, no water can be, but in the empty spaces, water can be.

The base is 7×7 , so area is 49 sq units.

Bottom layer cubes cover 25 sq units, so the area available for water in the bottom 1 unit height is $49 - 25 = 24$ sq units.

But this is only from $z=0$ to $z=1$, because above $z=1$, the cubes are only in certain places.

z from 0 to 1: bottom layer cubes occupy 25 sq units, so water area is 24 sq units.

Now, from $z=1$ to $z=2$: this is between first and second layer.

The second layer has 9 cubes. But the second layer is on top of the first layer, so at height $z=1$ to $z=2$, the cubes are present only where the second layer cubes are.

The second layer is placed on the first layer, so the cubes are from $z=1$ to $z=2$.

Similarly, bottom layer from $z=0$ to $z=1$.

Then second layer from $z=1$ to $z=2$, and top layer from $z=2$ to $z=3$.

The water height is 2 units from bottom, so water surface at $z=2$.

But at $z=2$, there is the top of the second layer and the bottom of the top layer, but since it's a point, we can think of the water level at $z=2$.

Now, when water is filled to height 2, that means from $z=0$ to $z=2$, the water is present, but in the regions not occupied by cubes.

So, to find water volume, I need the volume from $z=0$ to $z=2$ minus the volume occupied by cubes in that range.

Cubes below $z=2$ are the bottom layer and second layer, so $25 + 9 = 34$ cubes, each $1 \times 1 \times 1$, so volume 34 cubic units.

The total volume from $z=0$ to $z=2$ is the base area times height, $49 * 2 = 98$ cubic units.

But this includes the space occupied by cubes, so the actual space available for water is total volume minus cube volume in that region.

Volume from $z=0$ to $z=2$ is $49 * 2 = 98$ cu units.

Cubes in this region: bottom layer ($z=0$ to 1) and second layer ($z=1$ to 2), so $25 + 9 = 34$ cubes, volume 34 cu units.

Therefore, water volume = $98 - 34 = 64$ cubic units.

Is that right? But the water is only in the voids, so yes, that should be the water volume.

But let me confirm.

When water is filled to height 2, and since at $z=2$, the second layer top is at $z=2$, and water surface is at $z=2$, so it should be fine.

Now, the top layer is from $z=2$ to $z=3$, with one cube, but since water is only up to $z=2$, that cube is not submerged, so we don't care for now.

So water volume is 64 cubic units.

Now, the lid is closed, so when we invert, the water and air are trapped, but since it's inverted, the cubes will be in different positions.

Now, we invert the box upside down. So now, what was the top is now the bottom, and what was bottom is top.

But the base remains parallel to ground, so after inversion, the new bottom is what was the lid, and new top is what was the original bottom.

But the container itself is inverted, so the shape is the same, but the cubes inside are fixed, glued, so their positions relative to the container change? No.

The cubes are glued inside the container, so when we invert the container, the cubes also invert with it.

So, originally, with container upright: bottom at $z=0$, top at $z=3$.

Cubes: layer 1 from $z=0$ to 1, layer 2 from $z=1$ to 2, layer 3 from $z=2$ to 3.

After inversion, container upside down: let's say new coordinates, new bottom at $z'=0$, which was original $z=3$, and new top at $z'=3$, which was original $z=0$.

But since the container is inverted, the cubes are now upside down relative to the new orientation.

Originally, the stacking: bottom layer had 25 cubes, then 9 on top, then one on top.

After inversion, when we turn it upside down, what was the top layer is now at the bottom, and what was bottom is at top.

But the cubes are glued, so the relative positions are fixed, but the orientation changes.

Let me think carefully.

Suppose the container has a fixed coordinate system, but when we invert, we rotate it.

Perhaps it's better to think that after inversion, the new base is at the original top, and new top at original bottom.

The cubes are fixed in position relative to the container, so after inversion, the cubes that were near the original top are now near the new bottom.

Specifically: originally, layer 3 (one cube) was at $z=2$ to 3, layer 2 (9 cubes) at $z=1$ to 2, layer 1 (25 cubes) at $z=0$ to 1.

After inversion, new bottom is at original $z=3$, so at new $z'=0$, which is original $z=3$.

The cube that was at layer 3, from original $z=2$ to 3, is now at new height: since new $z'=0$ is original $z=3$, new $z'=1$ is original $z=2$, and so on.

Let's define the new height coordinate.

Set new z' such that $z' = 3 - z$, so when $z=0$, $z'=3$; $z=1$, $z'=2$; $z=2$, $z'=1$; $z=3$, $z'=0$.

After inversion, the new bottom is at $z'=0$ (original $z=3$), new top at $z'=3$ (original $z=0$).

Now, the cubes: originally, the layer 3 cube was from $z=2$ to 3. In new coordinates, $z=2$ to 3 corresponds to $z'=1$ to 0? Let's see:

When $z=2$, $z'=3-2=1$; $z=3$, $z'=0$. So z from 2 to 3, z' from 1 to 0, but since height, we think from bottom to top.

In new orientation, from new bottom $z'=0$ to top $z'=3$.

At new $z'=0$ (original $z=3$), it was the top of layer 3.

But layer 3 cube: originally from $z=2$ to $z=3$. So in new coordinates, when $z'=0$, it is original $z=3$; $z'=1$, original $z=2$; so the cube that was from $z=2$ to 3 is now from $z'=1$ to $z'=0$? That would be decreasing, but we should think of the interval.

The cube occupies the space that was between original $z=2$ and $z=3$. After inversion, that space is between new $z'=0$ and $z'=1$, because $z'=0$ is original $z=3$, $z'=1$ is original $z=2$, so the cube is from new $z'=0$ to $z'=1$.

Similarly, layer 2 cubes: originally from $z=1$ to $z=2$. Original $z=1$ to $z=2$ corresponds to new $z'=2$ to $z'=1$? $z=1$, $z'=2$; $z=2$, $z'=1$. So from $z'=1$ to $z'=2$? Let's list:

Original $z=0$: new $z'=3$

Original $z=1$: new $z'=2$

Original $z=2$: new $z'=1$

Original $z=3$: new $z'=0$

So, for a cube originally from $z=a$ to $a+1$, after inversion, it is from new $z' = (3-(a+1))$ to $z' = (3-a)$, but since z' increases upwards, we need min and max.

For layer 3: original $z=2$ to 3 , so min $z=2$, max $z=3$. New min $z' = 3 - \max z = 3-3=0$, new max $z' = 3 - \min z = 3-2=1$. So from new $z'=0$ to $z'=1$.

Similarly, layer 2: original $z=1$ to 2 , min $z=1$, max $z=2$. New min $z' = 3-2=1$, new max $z' = 3-1=2$, so from new $z'=1$ to $z'=2$.

Layer 1: original $z=0$ to 1 , min $z=0$, max $z=1$. New min $z' = 3-1=2$, new max $z' = 3-0=3$, so from new $z'=2$ to $z'=3$.

Therefore, after inversion:

Near new bottom ($z'=0$ to 1): what was the top layer, one cube.

From $z'=1$ to 2 : what was the second layer, 9 cubes.

From $z'=2$ to 3 : what was the bottom layer, 25 cubes.

Now, the base area is still $7 \times 7 = 49$ sq units.

We have water volume of 64 cubic units, and air above, but since lid is closed, when we invert, the water will flow, and we need to find the new water height from the new bottom.

After inversion, the new bottom is at $z'=0$.

We need to find the height from $z'=0$ to the water surface.

Now, similarly, the available space for water changes with height.

First, let's find where the water is.

The water volume is 64 cu units.

In the inverted position, the cubes are now: from $z'=0$ to 1: one cube (layer 3), so area occupied: since one cube, it covers 1 sq unit, so water area in this layer: $49 - 1 = 48$ sq units for z' from 0 to 1.

From $z'=1$ to 2: 9 cubes, so area occupied 9 sq units, water area $49 - 9 = 40$ sq units.

From $z'=2$ to 3: 25 cubes, area occupied 25 sq units, water area $49 - 25 = 24$ sq units.

Now, the water will fill from the bottom up, since we pour or after inversion, gravity, so water will be at the bottom.

So, we need to find up to what height the water rises, given 64 units volume.

Let h be the height from new bottom, $z'=0$.

We need to find h such that the water volume up to height h is 64.

But depending on h , the area changes.

The critical heights are at $z'=1$ and $z'=2$, where the cube layers change.

So, we need to consider cases based on where h is.

Case 1: if $h \leq 1$, then water only in the bottom layer, where water area is 48 sq units (since one cube).

Volume for height h : $48 * h$

Set equal to 64: $48h = 64$, $h = 64/48 = 4/3 \approx 1.333$, which is greater than 1, so not possible, since for $h \leq 1$, volume max is $48 * 1 = 48 < 64$, so water must be higher than $z'=1$.

Case 2: if $1 < h \leq 2$, then from $z'=0$ to 1: water area 48, volume $48 * 1 = 48$

From $z'=1$ to h : water area 40 (since from $z'=1$ to 2, area 40), and height $(h-1)$

So volume from $z'=1$ to h : $40 * (h-1)$

Total water volume: $48 + 40(h-1)$

Set equal to 64:

$$48 + 40(h-1) = 64$$

$$40(h-1) = 64 - 48 = 16$$

$$h-1 = 16/40 = 2/5 = 0.4$$

$$h = 1 + 0.4 = 1.4$$

Now, $h=1.4$, which is between 1 and 2, so this is possible.

But we have another layer above, but since $h=1.4 < 2$, we don't reach $z'=2$ yet.

But let's check if this satisfies.

Volume: from 0 to 1: $48 * 1 = 48$

From 1 to 1.4: height 0.4, area 40, volume $40 \times 0.4 = 16$

Total $48 + 16 = 64$, yes.

Now, is there another case? If $h > 2$, but let's see.

If $h > 2$, say up to $z' = 2$ and above.

First, from $z' = 0$ to 1: volume $48 \times 1 = 48$

From $z' = 1$ to 2: area 40, height 1, volume $40 \times 1 = 40$

Total up to $z' = 2$: $48 + 40 = 88$

But water volume is only 64, which is less than 88, so water would not reach $z' = 2$, since $64 < 88$, so h must be less than 2.

In fact, from above, $h = 1.4 < 2$.

So, the water height from new bottom is 1.4 units.

But 1.4 is $7/5$? $1.4 = 14/10 = 7/5$, and $7/5$ is one of the options.

But let me confirm.

$h = 1.4 = 7/5$? $7/5 = 1.4$, yes.

But the question is "height from the bottom of the water", I think it means the depth of water, from the bottom.

In the inverted position, from new bottom $z' = 0$.

So height is $7/5$ units.

But let me see the options: A) $3/2 = 1.5$, B) $5/3 \approx 1.666$, C) $6/5 = 1.2$, D) $7/5 = 1.4$, E) $8/3 \approx 2.666$

So D) $7/5$.

But is this correct?

I think I missed something.

In the upright position, when water is at height 2, and we have cubes up to second layer, but at height exactly 2, what is happening?

In my calculation, I assumed that from $z = 0$ to 2, volume 98, minus cubes 34, water 64.

But at $z = 2$, the water surface is at the level where the second layer top is, and the third layer is above, not in water.

Similarly, after inversion, at $z' = 1.4$, which is between $z' = 1$ and 2.

From $z' = 0$ to 1: one cube, so water can be around it.

From $z' = 1$ to 1.4: in the region where there are 9 cubes, so water can be between them.

In the volume calculation, I used the area, which is fine because the cubes are uniform, I think.

But let me think about the stacking.

In the upright position, bottom layer 25 cubes, which is 5×5 , so probably centered, but since the base is 7×7 , there is a border of 1 unit on each side, I assume, but not specified, but for area calculation, it doesn't matter, as long as the covered area is 25 sq units.

Similarly, second layer: 9 cubes, which is 3×3 , probably centered on top of the first layer.

The problem says: "stacked to be side-by-side and glued inside, starting from the base with 25 cubes in the first layer, then 9 cubes on them in the second layer, and finally one cube on top"

It doesn't specify the positions, but typically, the 9 cubes are on top of the 25, so they might be centered, but for the cross-sectional area, since the cubes are full height in their layers, the area available for water is constant in each height interval, regardless of position, because we're integrating over the area.

For example, from $z=0$ to 1, at any z in $[0,1]$, the cross-section has 25 squares covered by cubes, so $49-25=24$ open for water, similarly for other layers.

Similarly after inversion.

So I think it should be fine.

But in the inverted position, at the bottom, we have only one cube, so most of the base is open.

Then water height 1.4.

But let me confirm the water volume again.

Upright: water up to height 2.

Volume from 0 to 2: $49 * 2 = 98$

Cubes: bottom layer (0-1) and second layer (1-2), both fully below $z=2$, so $25 + 9 = 34$ cubes, each 1 cu unit, so 34 cu units occupied.

Thus water volume: $98 - 34 = 64$, yes.

After inversion: from new $z'=0$ to 1: one cube, so water area $49 - 1 = 48$

Volume for $[0,1]$: $48 * 1 = 48$

Then from $z'=1$ to h , with $h>1$, area 40 (since 9 cubes from $z'=1$ to 2)

Volume needed: $64 - 48 = 16$

So $40 * (h - 1) = 16$

$h - 1 = 16/40 = 2/5 = 0.4$

$h = 1 + 0.4 = 1.4 = 7/5$

Now, since $h=1.4 < 2$, we don't reach the top layer with 25 cubes, so no issue.

Therefore, height from bottom is $7/5$ units.

But the question says "its height from the bottom of the water", I think it means the water depth.

In the context: "what will be its height from the bottom of the water"

The phrasing: "its height from the bottom of the water"

Let me read the original.

"When the base of this container is parallel to the ground, its height from the bottom of the water is 2 units."

"its" refers to the container? Or the water?

It says: "its height from the bottom of the water"

This is ambiguous.

Let me read carefully.

"A quantity of water is poured into the container and the lid is closed. When the base of this container is parallel to the ground, its height from the bottom of the water is 2 units."

"its" probably refers to the container, but "height from the bottom of the water" doesn't make sense.

Perhaps it's "the height of the water from the bottom".

I think it's a translation issue or something.

In context, it should be the water height from the bottom.

Similarly, in the question: "what will be its height from the bottom of the water"

Again, "its" might be the container, but that doesn't make sense.

I think it's poorly phrased.

Probably, it means "the height of the water from the bottom" in both cases.

In the first part: when upright, water height from bottom is 2 units.

After inversion, water height from the new bottom.

I think that's standard.

In my calculation, I got $7/5$.

But let me see if there is a trick.

Another thing: when the box is inverted, and we have one cube at the bottom, which might affect how water fills, but since we're dealing with volumes and areas, it should be fine.

Perhaps the water level is not flat, but due to the cubes, but I think for this purpose, we assume the water surface is horizontal, so we can use the area method.

Maybe I need to consider the air or something, but I think not.

Another thought: when upright, water height 2, but at height exactly 2, there might be the top of the second layer cubes, and the water is at that level, but since the cubes are solid, water can be at $z=2$ around the cubes.

Similarly, after inversion, at $z'=1.4$, it's fine.

Perhaps the lid is closed, so when we invert, the air is also there, but since we're finding water height, and water volume is fixed, it should be ok.

I think my reasoning is correct.

But let me see the answer choices; $7/5$ is there.

But why are there other fractions? Maybe I missed something.

Perhaps in the upright position, when water is at height 2, and the second layer has 9 cubes, but the top layer has one cube above, but since water is at $z=2$, and the one cube is from $z=2$ to 3, so at $z=2$, it is the bottom of that cube, so water might be under it or something.

In the volume calculation, I didn't include the top cube because it's above water.

Similarly, after inversion.

In the inverted position, at the bottom, we have one cube, so from $z'=0$ to 1, water can be around it.

At $z'=1.4$, which is above $z'=1$, so in the region with 9 cubes.

Now, the 9 cubes are 3×3 , so there might be spaces between them, but in the area calculation, I used $49-9=40$, which is the open area, assuming that water can flow between cubes.

Similarly, in upright, it was fine.

Perhaps for the second layer, the 9 cubes are placed such that they are on the first layer, so the area might be different, but I think for the cross-section perpendicular to height, the area available is constant in each layer.

For example, between $z=1$ and $z=2$, at any height, the cross-section has 9 squares covered, so 40 open, regardless of position, since the cubes are full height in that interval.

Similarly for others.

So I think it's correct.

Perhaps the stacking: the 9 cubes are on top of the 25, so they might be centered, but the uncovered area is still 40 per unit height in that layer.

Similarly.

So I think $h=7/5$ is correct.

But let me calculate numerically.

Water volume: 64

After inversion:

Bottom layer: height 0 to 1, volume available: 48

But we need 64, so 48 used, remaining 16.

Next layer: height 1 to 2, area 40 per unit height, so for height h above 1, volume $40h$, set $40h=16$, $h=0.4$, so total height 1.4.

Yes.

If I think in terms of the container.

Total volume of container: $49 \times 3 = 147$

Cubes: 35, so total void volume: $147 - 35 = 112$

But when upright, water filled to height 2, void volume from 0 to 2: as before, $98 - 34 = 64$, or void: total void minus void above $z=2$.

Above $z=2$, from $z=2$ to 3, area 49, but with one cube, so void volume: $49 \times 1 - 1 = 48$, since one cube occupies 1.

Volume from $z=2$ to 3: $49 \times 1 = 49$, minus cube volume 1, so void 48.

Total void: 112, so void below $z=2$: $112 - 48 = 64$, same as before.

After inversion, void volume is still 112, but distributed differently.

Now, water volume 64, so it will fill part of the void.

In inverted position, void per layer:

From $z'=0$ to 1: area 48, void volume 48

From $z'=1$ to 2: area 40, void volume 40

From $z'=2$ to 3: area 24, void volume 24

Total void $48 + 40 + 24 = 112$, yes.

Now, water fills from bottom: first, fills the bottom void: 48, which is less than 64, so all of bottom void filled.

Remaining water: $64 - 48 = 16$

Now, this 16 must fill part of the middle void.

Middle void has volume 40, so it fills a height h such that $40 \times h = 16$, $h = 16/40 = 0.4$

So from $z'=1$ to $z'=1.4$, height 0.4.

Thus water height from bottom: 1.4

Since the bottom void is fully filled, and part of middle.

So water surface at $z' = 1.4$

Height is 1.4 units.

Now, $1.4 = 14/10 = 7/5$.

So answer should be $7/5$.

But let me see the options: D) $7/5$

But why is there E) $8/3$? Maybe if I mistaken something.

Perhaps when inverted, the one cube at bottom might not allow water to be under it or something, but since it's a cube, and the base is flat, water can be around it.

Similarly, in the middle layer, with 9 cubes, water can be between them.

I think it's fine.

Another thought: when the box is inverted, and we have one cube at the bottom, but since it's glued, and the base, water can cover the base except where the cube is.

Similarly.

Perhaps the "height from the bottom of the water" is misinterpreted.

But I think it's the depth.

In the question: "what will be its height from the bottom of the water"

I think "it" refers to the container, but that doesn't make sense.

Perhaps "the height of the water from the bottom".

I think we should go with that.

Maybe in some contexts, but I think $7/5$ is correct.

But let me see if the water height could be different.

Another point: when upright, water height 2, but at $z=2$, it is the interface, and the one cube is sitting on the second layer, so at $z=2$, the bottom of the top cube is at $z=2$, so water might be exactly at that level, but since it's a point, it's fine.

After inversion, similar.

Perhaps for the area, when we have multiple cubes, the effective area.

But I think it's ok.

I recall that in some problems, if the cubes are not full layers, but here they are stacked in layers.

The second layer has 9 cubes, which is less, so between $z=1$ and 2, the area is 40, but at $z=1$, it might be different, but since we integrate, it should be fine.

Similarly.

I think I should accept $7/5$.

But let me see the answer choices; there is $6/5=1.2$, which is less, and $5/3\approx 1.67$, larger.

If I had mistaken the layers.

Suppose in upright, when water is at height 2, and the second layer is from $z=1$ to 2, so at $z=2$, it is included, but the top cube starts at $z=2$, so if we think the water is up to but not including $z=2$, but since it's continuous, volume is the same.

Similarly.

Perhaps the first layer: bottom at $z=0$, but the base might have the bottom, but I think it's fine.

Another idea: when the box is inverted, the new bottom has only one cube, so the water might be deeper because less obstruction.

In upright, with 25 cubes at bottom, water area less, so for same volume, height higher, but here volume is fixed, and after inversion, bottom has more area open, so water height less, which is consistent, $1.4 < 2$.

Upright water height 2, inverted 1.4, less, makes sense.

If it were full, no cubes, water height would be $64/49 \approx 1.306$, but with cubes, it varies.

Upright: with cubes, to have 64 volume, since from 0 to 1, area 24, volume 24, then from 1 to h , area 40, set $24 + 40(h-1) = 64$, $40(h-1) = 40$, $h-1=1$, $h=2$, which matches given.

After inversion, as before.

I think it's correct.

So the height should be $7/5$ units.

But $7/5$ is 1.4, and in fractions.

So answer D) $7/5$

But let me see the options again.

A) $3/2=1.5$ B) $5/3\approx 1.667$ C) $6/5=1.2$ D) $7/5=1.4$ E) $8/3\approx 2.667$

So D.

Perhaps the problem is that when inverted, the water height is measured from the bottom, but in the cube region, it might be different, but I think not.

Maybe the "bottom of the water" means the lowest point of water, but I think it's the water surface height from container bottom.

I think I should go with that.

The transparent box has a base area of $7 \times 7 = 49$ square units and a height of 3 units. The 35 unit cubes are stacked such that the first layer (bottom) has 25 cubes, the second layer has 9 cubes, and the third layer (top) has 1 cube. The cubes occupy volumes in specific height intervals relative to the container's orientation.

Upright Position (Original Orientation)

Water height from the bottom is given as 2 units.

The volume from height 0 to 2 is $49 \times 2 = 98$ cubic units.

Cubes in this region: the first layer (height 0 to 1, 25 cubes) and the second layer (height 1 to 2, 9 cubes), totaling 34 cubes with a volume of 34 cubic units.

Water volume = total volume from 0 to 2 minus cube volume in this region: $98 - 34 = 64$ cubic units.

Inverted Position

After inversion, the container is upside down, with the new bottom at the original top (height 3) and the new top at the original bottom (height 0). The cubes are glued, so their positions relative to the container change:

New height z' from 0 to 1 (original height 3 to 2): contains what was the top layer (1 cube).

New height z' from 1 to 2 (original height 2 to 1): contains what was the second layer (9 cubes).

New height z' from 2 to 3 (original height 1 to 0): contains what was the first layer (25 cubes).

The cross-sectional area available for water in each interval is:

$z' = 0$ to 1: $49 - 1 = 48$ square units.

$z' = 1$ to 2: $49 - 9 = 40$ square units.

$z' = 2$ to 3: $49 - 25 = 24$ square units.

Water volume is 64 cubic units, and it fills from the new bottom ($z' = 0$) upward.

Volume from $z' = 0$ to 1: $48 \times 1 = 48$ cubic units (fully filled, as $48 < 64$).

Remaining water volume: $64 - 48 = 16$ cubic units.

This fills part of the interval $z' = 1$ to 2, where the area is 40 square units.

Total water height from new bottom: $1 + 0.4 = 1.4 = \frac{7}{5}$ units.

Since $1.4 < 2$, the water does not reach $z' = 2$, and the calculation is consistent.

Thus, the height from the bottom of the water after inversion is $\frac{7}{5}$ units.

$\boxed{\frac{7}{5}}$

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