Based on the typical life cycle of a butterfly, the egg stage lasts 10 days, followed by the caterpillar stage for 30 days, the chrysalis stage for 14 days, and the butterfly stage for 8 days. If a butterfly is in the caterpillar stage on the 1st of April, in the egg stage x days prior to that day, and in the butterfly stage 4x - 24 days after that day, what is the sum of the integer values that x can take? A)62, B)63, C)66, D)67, E)72

Answer from Mathful

Thought for 450 seconds

The problem describes the life cycle of a butterfly: egg stage lasts 10 days, caterpillar stage for 30 days, chrysalis stage for 14 days, and butterfly stage for 8 days. A butterfly is in the caterpillar stage on the 1st of April. It was in the egg stage x days prior to that day, and it will be in the butterfly stage 4x - 24 days after that day. I need to find the sum of the integer values that x can take.

First, I need to understand the timeline. Let's denote the 1st of April as day 0 for simplicity. So, on day 0, it's in the caterpillar stage.

It was in the egg stage x days prior to day 0. Since prior means before, that would be on day -x.

The egg stage lasts 10 days, so the butterfly was in the egg stage for 10 days ending at some point before it became a caterpillar.

I need to think about when each stage starts and ends.

Let me define the start of each stage.

Let T_egg be the time when the egg stage starts.

Egg stage lasts 10 days, so it ends at T_egg + 10.

Then caterpillar stage starts at T_egg + 10 and lasts 30 days, so it ends at T_egg + 40.

Chrysalis stage starts at T_egg + 40 and lasts 14 days, so it ends at T_egg + 54.

Butterfly stage starts at T_egg + 54 and lasts 8 days, so it ends at T_egg + 62.

Now, it's given that on day 0 (1st April), it's in the caterpillar stage. Caterpillar stage is from T_egg + 10 to T_egg + 40.

So, on day 0, it's in caterpillar, meaning:

$T\_egg + 10 \leq 0 \leq T\_egg + 40$

The caterpillar stage starts at T_egg + 10 and ends at T_egg + 40, so it includes times from T_egg + 10 inclusive to T_egg + 40 exclusive? Or inclusive?

Typically, in such problems, we assume that the stages are continuous, and the end time is when it transitions, so probably inclusive at start, exclusive at end, but to be safe, I'll consider the inequalities carefully.

Since it's a continuous time, but we're dealing with days, probably we need to consider integer days, as x is likely an integer.

The problem asks for sum of integer values, so x is integer, and days are discrete.

I'll assume that time is measured in days, and each stage transition happens at the end of the day or something. But to make it simple, perhaps I should think of the start and end times as inclusive.

Let me denote that for a stage of duration d, it occupies d consecutive days.

So, egg stage: 10 days, say days 1 to 10, but relative to start.

Set the start of egg stage as time t=0 for the egg.

But earlier I set T_egg.

Set T_egg as the time when egg stage begins.

Then:

Egg stage: from T_egg to T_egg + 9 (inclusive), if it lasts 10 days, so days 1 through 10, but if it starts at T_egg, it occupies days T_egg, T_egg+1, ..., T_egg+9.

Similarly for others.

Assume that on day D, if it's in a stage, it occupies that whole day.

So, the stage changes at the end of the day or beginning, but to avoid confusion, let's define the interval for each stage.

Let S_egg be the set of days when it's in egg stage, similarly for others.

But perhaps it's easier to work with the start times.

Given that on day 0, it's in caterpillar stage.

Caterpillar stage lasts from day C_start to day C_start + 29, if 30 days, so days C_start, C_start+1, ..., C_start+29.

Similarly, egg stage: days E_start to E_start + 9.

The stages are sequential, so E_start + 10 = C_start, because right after egg ends, caterpillar begins.

Similarly, C_start + 30 = P_start (chrysalis start), P_start + 14 = B_start (butterfly start).

The end of one stage is the start of the next, so no gap.

So, egg ends at E_start + 9 (if inclusive), but duration is 10 days, so from E_start to E_start+9, that's 10 days.

Then caterpillar starts at E_start + 10? That would be a gap.

I think I made a mistake.

If egg stage starts on day A, and lasts 10 days, then it occupies days A, A+1, A+2, A+3, A+4, A+5, A+6, A+7, A+8, A+9. So 10 days, ending at the end of day A+9.

Then caterpillar stage starts on the next day, which is day A+10.

Caterpillar lasts 30 days, so days A+10 to A+39 inclusive? Let's see: from A+10 to A+39 inclusive is (A+39) - (A+10) + 1 = 30 days, yes.

Similarly, chrysalis starts on day A+40, lasts 14 days, so days A+40 to A+53 inclusive.

Butterfly starts on day A+54, lasts 8 days, so days A+54 to A+61 inclusive.

Now, on day 0, it's in caterpillar stage. Caterpillar stage is from day A+10 to A+39 inclusive.

So, A+10 ≤ 0 ≤ A+39.

Since days are integers, 0 is within [A+10, A+39] inclusive.

So, A+10 ≤ 0 and 0 ≤ A+39.

A+10 ≤ 0 implies A ≤ -10.

0 ≤ A+39 implies A ≥ -39.

So, A is an integer such that -39 ≤ A ≤ -10.

Now, it was in the egg stage x days prior to day 0. Day 0 is 1st April.

Prior to day 0, so on day -x.

It was in egg stage on day -x.

Egg stage is from day A to day A+9 inclusive.

So, for it to be in egg stage on day -x, we have A ≤ -x ≤ A+9.

Also, it will be in butterfly stage 4x - 24 days after day 0. So on day 4x - 24.

Butterfly stage is from day A+54 to A+61 inclusive.

So, A+54 ≤ 4x - 24 ≤ A+61.

Now, I have A in terms of inequalities, and x is related.

Specifically, from the egg stage condition: A ≤ -x ≤ A+9.

This is a double inequality.

I can write it as $A \leq -x$ and $-x \leq A+9$.

Which implies $-x -9 \leq A$ and $A \leq -x$.

From $-x \leq A+9$, so $A \geq -x -9$.

And from $A \leq -x$.

So, $-x -9 \leq A \leq -x$.

But A is integer, so A must satisfy $-x -9 \leq A \leq -x$.

Also, from earlier, $-39 \leq A \leq -10$.

Now, also, for the butterfly stage: $A+54 \leq 4x - 24 \leq A+61$.

Let me denote $D = 4x - 24$, so on day D, it's in butterfly stage, so $A+54 \leq D \leq A+61$.

Now, $D = 4x - 24$.

Since A is between $-x -9$ and $-x$, I can express everything in terms of x.

Note that A is at least $-x -9$ and at most $-x$.

Now, the butterfly condition: $A+54 \leq 4x - 24 \leq A+61$.

Since A is close to $-x$, I can substitute.

Let me consider the inequalities.

Set $A = -x - k$, but perhaps better to use the bounds.

Since $A \leq -x$ and $A \geq -x -9$, and A is integer.

Now, the butterfly condition gives two inequalities:

$A + 54 \leq 4x - 24$

$4x - 24 \leq A + 61$

Let me write them as:

$A \leq 4x - 24 - 54 \Rightarrow A \leq 4x - 78$

And $A \geq (4x - 24) - 61 \Rightarrow A \geq 4x - 85$? Let me calculate.

From $4x - 24 \leq A + 61$, so $4x - 24 - 61 \leq A \Rightarrow A \geq 4x - 85$.

And from $A + 54 \leq 4x - 24$, so $A \leq 4x - 24 - 54 \Rightarrow A \leq 4x - 78$.

So we have:

$A \geq 4x - 85$

and

$A \leq 4x - 78$

But also, from egg stage, $A \ge -x - 9$ and $A \le -x$.

Moreover, $A \ge -39$ and $A \le -10$.

Now, since A must satisfy all these, we can combine the inequalities.

So, overall, A must satisfy:

$\max(-x-9, 4x - 85, -39) \le A \le \min(-x, 4x - 78, -10)$

And since A is integer, and also the inequalities must be consistent, meaning the lower bound ≤ upper bound.

Moreover, since A is determined by x, but for each x, A must be in that range, but actually for each x, there might be possible A, but in this case, since the egg stage condition already gives a range for A in terms of x, and the butterfly condition gives another constraint.

But actually, for a given x, the egg stage condition requires A to be between -x-9 and -x, inclusive, and integer.

Then for that A, the butterfly condition must hold, but since A is chosen based on x, we need that for the same A, both are satisfied, but actually, the butterfly condition gives another inequality that A must satisfy.

But since for each x, we can compute the required A from egg, but A is not fixed; for each x, there might be multiple A that satisfy the egg condition, but actually, for a fixed x, the egg condition gives a specific interval for A, and then the butterfly condition must hold for that A.

But A has to be the same for both, so for a given x, the A that satisfies the egg condition must also satisfy the butterfly inequalities.

Moreover, since the egg condition gives A in $[-x-9, -x]$, and butterfly requires $A \ge 4x - 85$ and $A \le 4x - 78$, so we need that the interval $[-x-9, -x]$ intersects with $[4x-85, 4x-78]$? No.

A has to satisfy both sets of inequalities simultaneously.

So, for a given x, A must be an integer such that:

max(lower bounds) ≤ A ≤ min(upper bounds)

The lower bounds are: $-x -9$, $4x - 85$, and -39

Upper bounds: $-x$, $4x - 78$, and -10

And since A must be in the intersection of all these inequalities.

Moreover, we have the additional constraint from the caterpillar stage that $A \ge -39$ and $A \le -10$, but since we already have it, we can include it.

So, for each integer x, we need that

$\max(L1, L2, L3) \le \min(U1, U2, U3)$

Where $L1 = -x -9$, $L2 = 4x - 85$, $L3 = -39$

U1 = -x, U2 = 4x - 78, U3 = -10

And then A is integer in that range, but actually for the existence, we need that the lower bound ≤ upper bound, and since A is chosen from a range, as long as the min upper bound is at least the max lower bound, and the range is non-empty, but since the range for A is continuous in a sense, but with integers, as long as min U ≥ max L, and since the interval length is at least 1, it should be ok, but actually in this case, because the egg condition requires A in [-x-9, -x], which has length 9, and we have additional constraints, but to ensure there is an integer A satisfying all, we need that the max of lower bounds is less than or equal to the min of upper bounds.

Moreover, since the range might be tight, but as long as max L ≤ min U, and since the bounds might not be integer, but since A is integer, we need floor(min U) ≥ ceil(max L), but to simplify, since the differences are large, probably max L ≤ min U is sufficient for existence of integer A, but we should check.

In this case, the egg condition alone gives A in an interval of length 9, so likely for most x where the inequalities hold, there will be an A, but we need to ensure.

To be precise, we need that there exists integer A such that:

A ≥ -x -9

A ≥ 4x - 85

A ≥ -39

A ≤ -x

A ≤ 4x - 78

A ≤ -10

And A integer.

But since A ≤ -x and A ≥ -x -9, and the interval length is 10? From -x-9 to -x inclusive, so number of integers is 10? From a to b inclusive, number is b-a+1.

Here from -x-9 to -x inclusive, so (-x) - (-x-9) + 1 = 9 +1 =10 integers.

Similarly, other constraints might cut it down, but as long as the inequalities are satisfied, there should be an A.

The critical part is that the lower bounds are all ≤ the upper bounds.

Specifically, we need:

max(-x-9, 4x-85, -39) ≤ min(-x, 4x-78, -10)

And also since A must be in caterpillar range, which is already included.

Now, min(-x, 4x-78, -10) and max of the lowers.

Since we have multiple inequalities, we need to consider all pairwise inequalities to ensure consistency.

The binding constraints might come from the butterfly and egg conditions.

I recall that we have $A \geq -x -9$ and $A \leq -x$, and $A \geq 4x -85$ and $A \leq 4x -78$.

Since A is approximately -x, and 4x - something, we can set up the relation between the inequalities.

Consider the difference between upper and lower bounds.

Set the two expressions for A.

From egg, A is around -x, from butterfly, A is around 4x - something.

So, set $-x \approx 4x - c$ for some c.

So $5x \approx c$, $x \approx c/5$.

But let's write the inequalities we must have.

First, since $A \leq -x$ and $A \geq 4x -85$, so $-x \geq 4x -85$, because $A \leq -x$ and $A \geq 4x -85$, so $-x \geq 4x -85$.

Similarly, other combinations.

Let me list the necessary conditions.

From $A \leq -x$ and $A \geq 4x -85$, so $-x \geq 4x -85 \Rightarrow -x -4x \geq -85 \Rightarrow -5x \geq -85 \Rightarrow$ multiply by -1, reverse inequality, $5x \leq 85 \Rightarrow x \leq 17$.

From $A \leq -x$ and $A \geq -x -9$, but that's always true in the range, since it's the same as the egg condition.

We need inequalities that must hold regardless of which bounds are active.

Another one: from $A \geq -x -9$ and $A \leq 4x -78$, so $-x -9 \leq 4x -78 \Rightarrow -x -4x \leq -78 +9 \Rightarrow -5x \leq -69 \Rightarrow$ multiply by -1, reverse, $5x \geq 69 \Rightarrow x \geq 13.8$

$69/5 = 13.8$, so $x \geq 13.8$, since x integer, $x \geq 14$.

We have more.

Also, from $A \geq 4x -85$ and $A \leq -x$, which we did, $x \leq 17$.

But also, we have the other bounds.

For example, $A \leq -10$ and $A \geq -39$, but we'll see.

Also, $A \geq 4x -85$ and $A \leq 4x -78$, which implies that $|A - (4x - 81.5)| \leq 3.5$ or something, but perhaps not necessary.

Now, also, since $A \leq \min(-x, 4x-78, -10)$ and $A \geq \max(-x-9, 4x-85, -39)$

Now, we also need to satisfy with the min and max.

But we have $x \geq 14$ and $x \leq 17$ from above.

$x \geq 14$ and $x \leq 17$, and x integer, so x=14,15,16,17.

But we have more constraints because of the other bounds, like -10 and -39.

Moreover, we need to ensure that the max lower bound ≤ min upper bound.

So let's check for each x from 14 to 17.

First, recall that A must also be in [-39,-10].

Now, for each x, we can compute the required A range.

Start with x=14.

Then, from egg stage: $A \geq -x - 9 = -14-9 = -23$

$A \leq -x = -14$

So A in [-23,-14] inclusive.

Now, from butterfly: $A \geq 4x - 85 = 4*14 - 85 = 56-85 = -29$

$A \leq 4x - 78 = 56-78 = -22$

Butterfly requires $A \geq 4x-85 = -29$ and $A \leq 4x-78 = -22$

Now, also, $A \geq -39$, but $-23 > -39$, so max lower is max(-23, -29, -39) = -23? No.

A must be ≥ max of the lower bounds: max(-x-9, 4x-85, -39) = max(-23, -29, -39) = -23 (since $-23 > -29 > -39$)

And ≤ min of upper bounds: min(-x, 4x-78, -10) = min(-14, -22, -10)

Now min of -14, -22, -10 is -22? No, min of numbers: the smallest value.

min(-14, -22, -10) = -22, since $-22 < -14 < -10$.

But now, lower bound is -23, upper bound is -22.

So $-23 \leq A \leq -22$

And A integer, so A = -23 or -22.

Now, also, A must be ≤ -10? From the caterpillar constraint, but -23 and -22 are both less than -10? $-23 < -10$, yes, but the constraint is $A \leq -10$? No.

Earlier I set $A \leq -10$ from caterpillar stage? Let me check.

From caterpillar on day 0, $A \geq -39$ and $A \leq -10$.

$A \leq -10$? Earlier: $A \geq -39$ and $A \leq -10$, but $A \leq -10$? No.

Recall: earlier, for caterpillar, $A+10 \leq 0 \leq A+39$, so $A \leq -10$ and $A \geq -39$? $A+10 \leq 0$ implies $A \leq -10$, and $0 \leq A+39$ implies $A \geq -39$.

But $A \leq -10$ and $A \geq -39$.

But now for x=14, we have $A \leq -22$ and $A \geq -23$, so possible A=-23 or -22.

But also A must be ≤ -10? -23 ≤ -10 is true, but the constraint is A ≤ -10? No, the constraint is A ≤ -10? From caterpillar, yes, A ≤ -10.

But -23 ≤ -10 is true, but A ≤ -10 means A is at most -10, which includes -23, -22, etc., as long as ≤ -10.

But here A is -23 or -22, both ≤ -10? -23 < -10, yes, but the issue is that A must be ≤ -10, but -23 and -22 are both less than -10, so that's fine? But we also have the butterfly upper bound is -22, and lower is -23.

But A must be ≤ -10, and -22 ≤ -10 is true, so no problem? But we need to check if it satisfies all.

A=-23 or -22.

But we have another upper bound: min includes -10, but since -22 < -10, min is -22.

Now, but A must also be ≤ -10? From caterpillar, A ≤ -10, but -22 ≤ -10 is true, so it's satisfied.

Similarly, A ≥ -39, -23 ≥ -39, yes.

But now, we need to check if for this A, the stages make sense, but probably.

But we have min upper bound is min(-14, -22, -10) = min of these values: -22 is the smallest, so min is -22.

Max lower bound is max(-23, -29, -39) = -23.

So $-23 \leq A \leq -22$.

A integer, so A=-23 or A=-22.

Now, but we also have the egg condition: for each A, but for a fixed x, we have A must be in [-23,-14], and here -23 and -22 are both in this interval? -23 ≥ -23 and ≤ -14? -23 ≤ -14 yes, but -23 is included.

[-23,-14] includes -23 and -22, yes.

Now, but we need to verify the butterfly stage day.

The butterfly condition is on day D=4x-24=4*14-24=56-24=32.

It should be in butterfly stage on day 32.

Butterfly stage is from A+54 to A+61 inclusive.

For A=-23, butterfly from -23+54=31 to -23+61=38, so days 31 to 38.

Day 32 is within 31 to 38? Yes, since 31≤32≤38.

For A=-22, butterfly from -22+54=32 to -22+61=39, so days 32 to 39, includes 32.

Now, also, it was in egg stage on day -x = -14.

Egg stage: from A to A+9.

For A=-23, days -23 to -14? A+9=-23+9=-14, so days -23,-22,...,-14.

Day -14 is included? From -23 to -14 inclusive: -23,-22,...,-14. That's 14-(-23)+1=14+23+1=38 days? No.

Number of days: from -23 to -14 inclusive: values -23,-22,...,-14. First term -23, last -14, number = (-14) - (-23) + 1 = -14+23+1=10, yes.

Day -14 is included.

Now, it was in egg stage on day -x = -14, which is the last day of egg stage for A=-23.

Similarly, for A=-22, egg stage from -22 to -13? A+9=-22+9=-13.

So days -22 to -13 inclusive.

Day -14: is -14 in this range? -22 ≤ -14 ≤ -13? -14 is greater than -22 but less than -13? -14 < -13, so -14 < -13, so -14 is not ≥ -22 and ≤ -13? -22 ≤ -14 is true, but -14 ≤ -13? -14 ≤ -13 is true, since -14 is less than -13.

Inclusive: if days are integers, -14 is included if the lower bound is ≤ -14 and upper bound ≥ -14.

For A=-22, egg from -22 to -13 inclusive.

-22 ≤ -14? Yes, and -14 ≤ -13? Yes, so -14 is included? But -14 is between -22 and -13, yes, since -22 < -14 < -13? -14 is greater than -22 and less than -13, but -13 is larger than -14? No.

Numerically: -22 < -14 < -13? Is -14 less than -13? Yes, -14 < -13, so -14 is greater than -22 but less than -13, and since it's inclusive, if the set is [-22,-21,...,-13], does it include -14? Yes, because -14 is an integer in that range.

The days are A to A+9 inclusive, so for A=-22, days -22, -21, -20, -19, -18, -17, -16, -15, -14, -13. So yes, includes day -14.

Similarly for both.

Now, on day 0, caterpillar stage.

For A=-23, caterpillar starts at A+10= -23+10= -13, ends at A+39= -23+39=16? Earlier I said caterpillar from A+10 to A+39 inclusive.

A+10=-13, A+39=16, so days -13 to 16 inclusive.

Day 0 is between -13 and 16, yes.

Similarly for A=-22, caterpillar from -22+10= -12 to -22+39=17, so days -12 to 17, includes 0.

Now, the butterfly on day 32, as above, for both A, it's included.

But we have another constraint: A must be ≤ -10? From caterpillar, A ≤ -10.

A=-23 < -10, A=-22 < -10, both satisfy.

But earlier I thought A ≤ -10 is the constraint, but actually in the min upper bound, we have min(-x, 4x-78, -10) = min(-14,-22,-10)= -22 for x=14.

But A ≤ -22 from this min, but the caterpillar constraint is A ≤ -10, which is less restrictive since -22 < -10, but the min already gives A ≤ -22, which is stronger.

But for A, it must satisfy A ≤ -10, but since A ≤ -22 < -10, it's automatically satisfied.

However, for x=14, we have possible A=-23 and -22, both seem to work.

But is there a problem? The butterfly stage on day 32 is fine, egg on day -14 is fine, caterpillar on day 0 is fine.

But the problem is that for each x, we might have multiple A, but the question is about x, and sum of x that work, but here for x=14, it works with both A.

But probably it's fine, as long as there is some A that makes it work.

The condition is that the butterfly is in butterfly stage 4x-24 days after, and was in egg x days prior, and on day 0 caterpillar.

So for each such x, as long as there is an A that satisfies all, it should be ok.

Now, but we have x=14, and it seems to work.

But let me check the value.

x=14, D=4*14-24=56-24=32, as above.

Now, for x=15.

x=15.

Egg condition: A ≥ -15 -9 = -24

A ≤ -15

So A in [-24,-15] inclusive.

Butterfly: A ≥ 4*15 -85=60-85= -25

A ≤ 4*15 -78=60-78= -18

Now, max lower bounds: max(-24, -25, -39) = -24 (since -24 > -25)

Min upper bounds: min(-15, -18, -10) = min(-15,-18,-10)= -18? Since -18 < -15 < -10.

So -24 ≤ A ≤ -18

Now, also A must be in [-39,-10], which is satisfied.

A integer, so A= -24,-23,-22,-21,-20,-19,-18? From -24 to -18 inclusive.

But we also have the caterpillar constraint A ≤ -10, which is true, but also from the min upper bound, A ≤ -18, but in the range, A can be up to -18.

But we need to check if this A satisfies the egg condition.

Egg condition is A in [-24,-15], and here A from -24 to -18, all are ≤ -15? -18 ≤ -15 yes, and ≥ -24, so yes, all are within [-24,-15].

Now, butterfly on day D=4*15-24=60-24=36.

Butterfly stage: from A+54 to A+61.

For A in -24 to -18.

A+54: -24+54=30, -18+54=36

A+61: -24+61=37, -18+61=43

So butterfly days: for each A, from max(A+54,30) to min(A+61,43), but actually for each A, the range is fixed.

For example, A=-24, butterfly days -24+54=30 to -24+61=37, so 30 to 37.

Day 36 is included? 30≤36≤37 yes.

A=-23, 31 to 38, includes 36.

A=-22,32 to 39, includes 36.

A=-21,33 to 40, includes 36.

A=-20,34 to 41, includes 36.

A=-19,35 to 42, includes 36.

A=-18,36 to 43, includes 36.

So all good.

Now, egg on day -x=-15.

Egg stage: A to A+9.

For A=-24, days -24 to -15, includes -15.

A=-23, -23 to -14, includes -15? -23 ≤ -15 ≤ -14? -15 ≥ -23 and -15 ≤ -14? -15 ≤ -14 yes, so yes.

Similarly for others, since A ≤ -18? A is at least -24, at most -18, and day -15 is greater than -24 and less than -18? -24 ≤ A ≤ -18, so A+9 ≤ -18+9=-9, and A ≥ -24, so the egg days include from at least -24 to at most -9, so day -15 is always included since -24 ≤ -15 ≤ -9? -15 ≤ -9 is true, but for each A, the upper bound is A+9, which for A≥-24, A+9≥-15? A≥-24, A+9≥-15, but we need A+9 ≥ -15? Actually, to include day -15, we need A ≤ -15 and A+9 ≥ -15.

A ≤ -15 is true for all A in [-24,-18], since -18 ≤ -15? -18 < -15, yes, so A ≤ -18 < -15, so A ≤ -15.

And A+9 ≥ -24+9=-15? For A≥-24, A+9≥-15, but when A=-24, A+9=-15, so the upper bound is -15, so day -15 is included.

For other A, since A > -24, A+9 > -15, so day -15 is included as long as A ≤ -15, which is true.

So all good.

Now, for x=16.

x=16.

Egg: A ≥ -16-9= -25

A ≤ -16

So A in [-25,-16]

Butterfly: A ≥ 4*16-85=64-85= -21

A ≤ 4*16-78=64-78= -14

Now, max lower bounds: max(-25, -21, -39) = max(-25,-21,-39)= -21? Since -21 > -25

Min upper bounds: min(-16, -14, -10) = min(-16,-14,-10)= -16? Since -16 < -14 < -10.

So max lower = -21, min upper = -16.

So -21 ≤ A ≤ -16

Now, also A in [-39,-10], which is ok.

Now, A must be integer, so A= -21,-20,-19,-18,-17,-16.

Now, egg condition: A in [-25,-16], and -21 to -16 are all in this range? -21 ≥ -25 yes, -21 ≤ -16 yes, so yes.

Butterfly on day D=4*16-24=64-24=40.

Butterfly: A+54 to A+61.

A from -21 to -16.

A+54: -21+54=33, -16+54=38

A+61: -21+61=40, -16+61=45

So for A=-21, butterfly 33 to 40, includes 40.

A=-20,34 to 41, includes 40.

...

A=-16,38 to 45, includes 40. All good.

Egg on day -16.

For each A, egg days A to A+9.

A ≥ -21, so A+9 ≥ -12, but we need day -16 included.

A ≤ -16 and A+9 ≥ -16.

A ≤ -16 is true for all, since max A is -16.

A+9 ≥ -16? A ≥ -25, but we have A ≥ -21, so for A ≥ -21, A+9 ≥ -12 > -16? -12 > -16, so A+9 ≥ -12 ≥ -16, so yes, but we need specifically that -16 is in [A, A+9], so A ≤ -16 and A+9 ≥ -16.

A ≤ -16 is true, and A+9 ≥ -16 implies A ≥ -25, which is true since A ≥ -21 > -25? -21 > -25 yes, but A ≥ -21, so A+9 ≥ -12, which is greater than -16, so A+9 ≥ -12 > -16, so yes, A+9 ≥ -16 is satisfied.

Moreover, since A ≤ -16, and A+9 ≥ -12 > -16, but we need A+9 ≥ -16, which is true, and A ≤ -16, so the lower bound is satisfied, and since A+9 ≥ -16, and A ≤ -16, so -16 is at least A and at most A+9? Specifically, since A ≤ -16 and A+9 ≥ -16, so -16 is ≥ A and ≤ A+9, yes, so included.

For example, A=-21, egg days -21 to -12, includes -16.

A=-16, egg days -16 to -7, includes -16. Good.

Now, for x=17.

x=17.

Egg: A ≥ -17-9= -26

A ≤ -17

So A in [-26,-17]

Butterfly: A ≥ 4*17-85=68-85= -17

A ≤ 4*17-78=68-78= -10

Now, max lower bounds: max(-26, -17, -39) = max(-26,-17,-39)= -17? Since -17 > -26.

Min upper bounds: min(-17, -10, -10) = min(-17,-10,-10)= -17? Since -17 < -10.

So -17 ≤ A ≤ -17.

So A = -17.

Now, check if satisfies.

A = -17.

Egg: A to A+9 = -17 to -8, so days -17 to -8.

Day -x = -17, which is included, since it starts at -17.

Caterpillar: A+10= -17+10= -7 to A+39= -17+39=22, so days -7 to 22, includes day 0.

Butterfly: from A+54= -17+54=37 to A+61= -17+61=44.

Day D=4x-24=4*17-24=68-24=44.

Is 44 in butterfly? Butterfly days 37 to 44 inclusive, so yes, since it ends at 44, so day 44 is included.

Now, is there any other constraint? $A=-17$, and caterpillar requires $A \le -10$, $-17 \le -10$ yes, and $A \ge -39$ yes.

So all good.

Earlier I had $x=14,15,16,17$.

But for each, it works.

The sum would be $14+15+16+17=$ let's see, $14+17=31$, $15+16=31$, so 62.

And option A is 62.

But the problem says "sum of the integer values that x can take", and I have four values:14,15,16,17.

But in $x=14$, we had two possible A, but x is still the same, so it should be counted once.

Similarly for others.

But is there any x missing or extra?

Earlier I thought $x\ge14$ and $x\le17$, but let me check $x=13$.

I had from necessary conditions, $x\ge13.8$, so $x\ge14$, but let me verify $x=13$.

$x=13$.

Egg: $A \ge -13-9= -22$

$A \le -13$

So A in [-22,-13]

Butterfly: $A \ge 4*13-85=52-85= -33$

$A \le 4*13-78=52-78= -26$

Now, max lower bounds: $\max(-22, -33, -39) = -22$? $\max(-22,-33,-39)= -22$

Min upper bounds: $\min(-13, -26, -10) = \min(-13,-26,-10)= -26$? Since $-26 < -13 < -10$.

So max L = -22, min U = -26.

Is $-22 \le -26$? No, $-22 > -26$, so max L > min U, so no solution.

Indeed, lower bound -22, upper bound -26, but $-22 > -26$, so no A can satisfy $A \ge -22$ and $A \le -26$, impossible.

Now, what about $x=18$?

$x=18$.

Egg: $A \ge -18-9= -27$

$A \le -18$

Butterfly: $A \geq 4*18-85=72-85= -13$

$A \leq 4*18-78=72-78= -6$

Now, max lower: max(-27, -13, -39) = -13? Since -13 > -27

Min upper: min(-18, -6, -10) = min(-18,-6,-10)= -18? Since -18 < -10 < -6? -18 is smallest.

So $-13 \leq A \leq -18$? Is $-13 \leq -18$? No, -13 > -18, so again max L > min U, no solution.

Now, but earlier for x=17, it worked.

But in the options, 62 is there, but let me see if for all x from 14 to 17, it works, and sum is 62.

But in x=15, we had multiple A, but that's fine.

The problem is, in the butterfly stage condition, for some x, it might not be satisfied, but in this case it was.

But in x=14, when A=-23, butterfly is from 31 to 38, day 32 is included.

A=-22, 32 to 39, includes 32.

But is there a constraint I missed?

Another thing: the stages must be sequential and no overlap, but in this setup, since we defined sequentially, it should be fine.

Now, but the chrysalis stage: I didn't check, but probably not necessary, since we have the butterfly start after chrysalis.

But let me verify the timeline for one of them.

Take x=14, A=-23.

Egg: day A to A+9 = -23 to -14

Then caterpillar: starts at end of egg, so day A+10= -13 to A+39=16? -23+39=16, yes.

Chrysalis: starts at end of caterpillar, so day A+40= -23+40=17 to A+53= -23+53=30

Butterfly: day A+54=31 to A+61=38

Now, on day 0, caterpillar: day -13 to 16, includes 0, good.

x days prior: day -14, which is in egg, since egg ends at -14, so if inclusive, it is included.

4x-24=32, butterfly from 31 to 38, includes 32, good.

Now for A=-22 with x=14.

Egg: -22 to -13

Caterpillar: -12 to 17

Chrysalis: 18 to 31? A+40=-22+40=18, A+53=31

Butterfly: 32 to 39

Day 0 in caterpillar, good.

Day -14: egg days -22 to -13, includes -14? Yes, as before.

Day 32 in butterfly, good.

Now for x=15, A could be from -24 to -18, all work.

But in particular, for each x, there is at least one A that works, so x is possible.

Now sum is 14+15+16+17.

14+17=31, 15+16=31, total 62.

But the option is A)62, but let me see the answer choices: A)62, B)63, C)66, D)67, E)72.

So 62 is an option.

But is there x=13? We saw no.

x=18? No.

But earlier I assumed that the min and max bounds are sufficient, but for some x, even if max L ≤ min U, there might be no integer A if the bounds are not integer, but in this case, for all these x, the bounds are integers.

For x=14, max L=-23, min U=-22, so A=-23,-22.

x=15, max L=-24, min U=-18? Earlier for x=15, max lower was -24? No.

For x=15:

Max lower: max(-x-9, 4x-85, -39)=max(-24,-25,-39)= -24? Earlier I said max is -24.

Butterfly lower is 4x-85= -25, which is less than -24, so max is -24.

Min upper: min(-x,4x-78,-10)=min(-15,-18,-10)= -18? -18 is the smallest.

So -24 ≤ A ≤ -18.

All integers, no issue.

Similarly for others.

Now, but for x=17, A=-17 only.

But sum is 14+15+16+17=62.

But the problem says "the sum of the integer values that x can take", and I have four values.

But is there any constraint I missed?

In the butterfly stage, it says "the butterfly stage for 8 days", and I assumed it lasts 8 days, from day A+54 to A+61 inclusive, which is 8 days: 54,55,56,57,58,59,60,61? A+54 to A+61 inclusive: let k=A+54, then k to k+7 inclusive? (k+7)-k+1=8 days, yes.

Similarly for others.

Now, another thing: when it says "in the egg stage x days prior", and "in the butterfly stage 4x-24 days after", and we assumed that on that day, it is in that stage.

But for the egg stage on day -x, we assumed it is included, which it is.

But in the case where it is the boundary, like when it transitions, but in this case, since we defined inclusive, and transitions happen at the end of the day, so on the last day of egg, it is still egg, next day caterpillar.

Similarly for others.

For example, in x=17, day -17 is the first day of egg, so it's in egg.

Day 44 is the last day of butterfly? For A=-17, butterfly to A+61=-17+61=44, so day 44 is included, and since it ends at day 44, probably the next day it dies or something, but the stage lasts 8 days, so if it starts at day 37, ends at day 44, that's 8 days: 37,38,39,40,41,42,43,44. Yes.

Now, but the problem is, for some A, on the transition day, but in this case, we don't have transition on the days we care about, except possibly.

For example, in x=14, when A=-23, on day -14, it is the last day of egg, so it is in egg stage.

On day 32, for A=-23, butterfly from 31 to 38, day 32 is in butterfly.

Similarly.

Now, but in the caterpillar stage on day 0, for all, it's in caterpillar.

Now, but I recall that in the life cycle, after butterfly, it dies or something, but the problem doesn't specify, but since we are checking the day, and it's within the stage, should be fine.

Now, sum is 14+15+16+17=62.

But let me calculate: 14+17=31, 15+16=31, total 62.

14+15=29, 16+17=33, 29+33=62.

Now, option A is 62.

But the problem might have a trick, because in the butterfly stage, it says "4x - 24 days after that day", and "that day" is 1st April, when it's caterpillar.

Now, for x=14, 4x-24=32, so 32 days after 1st April.

But we need to ensure that this is within the butterfly stage, which it is.

But also, the butterfly stage might be after chrysalis, which it is.

But another thing: the total life span.

For example, for A=-23, butterfly ends at day 38, and day 32 is before that.

But all good.

But why is there an option 63? Maybe I missed x=13 or 18, but we saw not.

Perhaps for some x, no A satisfies, but in this case all do.

In x=15, we have A from -24 to -18, but all work, so x=15 is valid.

But perhaps the problem implies that there is exactly one such life cycle or something, but it doesn't say; it just says "a butterfly", and for each x, as long as there is a possible start time A that makes it work, then x is possible.

Moreover, for different x, different A, so each x corresponds to different scenarios.

Now, but notice that for different x, the A might overlap, but that's fine.

Perhaps for some x, the day 4x-24 is negative or something, but in this range, x≥14, 4x-24≥4*14-24=56-24=32>0, so after, good.

x days prior is -x, which is negative, but we handled.

Now, but let me check for x=17, we have A=-17, and it works.

Sum is 62.

But I recall that in the caterpillar stage, it lasts 30 days, and we have it on day 0.

Now, the prior day, day -1, for example, but not relevant.

Perhaps the issue is that the butterfly stage after might be too late, but in this case it's fine.

Another thought: in the expression "4x - 24", and it must be a positive integer, which it is for x≥6, but in our case x≥14>6.

Now, but let me see if for x=14, with A=-23, is there a problem with chrysalis or something.

Chrysalis: for A=-23, starts at A+40=17, ends at 30.

On day 0, it's caterpillar, which is before chrysalis, good.

Butterfly after, good.

The day it is in butterfly is day 32, which is after chrysalis, good.

Now, but the problem might be that for the butterfly stage to be after the caterpillar, which it is, but in this timeline, it is sequential.

Now, but in the life cycle, it goes egg, caterpillar, chrysalis, butterfly, so after caterpillar is chrysalis, then butterfly.

In our case, for each A, it follows that order.

For example, for x=14, A=-23:

Egg: -23 to -14

Caterpillar: -13 to 16

Chrysalis: 17 to 30

Butterfly: 31 to 38

All consecutive, no gap.

Similarly for others.

Now, for x=15, say A=-24:

Egg: -24 to -15

Caterpillar: -14 to 15? A+10=-14, A+39=15? -24+39=15, yes.

Chrysalis: 16 to 29? A+40=16, A+53=29

Butterfly: 30 to 37

On day 0: caterpillar -14 to 15, includes 0.

Day -15: egg -24 to -15, includes -15 (last day).

Day 36: butterfly 30 to 37, includes 36.

Good.

Now, for x=17, A=-17:

Egg: -17 to -8

Caterpillar: -7 to 22

Chrysalis: 23 to 36? A+40=23, A+53=36

Butterfly: 37 to 44

Day 0: caterpillar, good.

Day -17: egg, first day.

Day 44: butterfly, last day.

Good.

Now, all seem fine.

Sum is 14+15+16+17=62.

But why is there option 63? Maybe I need to include x=13 or 18, but they don't work.

Perhaps for some x, there is another possibility.

Or perhaps I missed that A must be such that the stages are within reasonable, but no.

Another thing: in the butterfly condition, it says "in the butterfly stage 4x-24 days after that day", and "that day" is when it's caterpillar, so the number of days after must be positive, which it is, since x≥14, 4x-24≥32>0.

But for the prior, it's negative, but we handled.

Perhaps the expression 4x-24 must be integer, but x is integer, so it is.

Or perhaps x must be such that 4x-24 is non-negative, which it is.

But let me check x=12, though we know it shouldn't work.

x=12.

Then egg: A≥-12-9=-21, A≤-12

Butterfly: A≥4*12-85=48-85=-37, A≤4*12-78=48-78=-30

Max lower: max(-21,-37,-39)=-21

Min upper: min(-12,-30,-10)=-30? min(-12,-30,-10)=-30

-21 ≤ A ≤ -30? -21 > -30, impossible.

Similarly.

Now, but in x=14 to 17, sum is 62.

But let me calculate the sum: 14+15+16+17.

14+15=29, 29+16=45, 45+17=62, yes.

Now, but option B is 63, which is close, maybe I missed x=13.5 or something, but x must be integer.

Perhaps for some x, the butterfly day is not within, but we checked.

Another thought: when it is in chrysalis stage, but the problem doesn't mention it, so probably not relevant.

Perhaps the total life span.

Or perhaps that the butterfly stage after caterpillar, but there is chrysalis in between, but in the timeline, from caterpillar to butterfly, there is chrysalis, but the number of days: from end of caterpillar to start of butterfly is the chrysalis stage.

But in our calculation, for example, on day 0 caterpillar, then chrysalis starts later, then butterfly.

The butterfly stage is 4x-24 days after day 0, which is after the caterpillar, and since chrysalis is in between, but as long as it's after, and we are in butterfly, it should be fine, as we checked.

But let me compute the time from caterpillar to butterfly.

The end of caterpillar is at A+39.

Start of butterfly is at A+54.

So the time between end of caterpillar and start of butterfly is (A+54) - (A+39) = 15 days, which is exactly the chrysalis duration? Chrysalis is 14 days? Contradiction?

No.

Caterpillar ends at A+39? Earlier I set caterpillar from A+10 to A+39 inclusive.

So the last day of caterpillar is day A+39.

Then chrysalis starts the next day, day A+40.

Chrysalis lasts 14 days, so from A+40 to A+53 inclusive? A+40 to A+53 inclusive is 14 days: (53-40)+1=14.

Then butterfly starts day A+54.

But the end of caterpillar is day A+39, start of chrysalis is day A+40, so there is a day in between? Day A+39 is caterpillar, day A+40 is chrysalis.

So no gap.

Similarly, chrysalis ends at day A+53, butterfly starts at A+54.

Now, the butterfly stage is on day D=4x-24.

In terms of after caterpillar: the day after caterpillar ends is day A+40, which is chrysalis start.

Butterfly starts at A+54.

The earliest day for butterfly is A+54.

On day 0, which is in caterpillar, so 0 ≤ A+39, and 0 ≥ A+10.

The butterfly stage starts at A+54.

The number of days after day 0 to reach butterfly start is (A+54) - 0 = A+54.

But we are not necessarily at the start; we are in the butterfly stage, which could be any day during it.

The earliest possible day for butterfly is A+54, latest is A+61.

Since A ≥ -39, A+54 ≥ 15.

A ≤ -10, A+54 ≤ 44.

In our case, D=4x-24, for x=14, D=32, which is greater than 15, but we need to ensure it's at least A+54, but since A ≤ -x, and x≥14, A≤-14, so A+54≤40, and D=32 for x=14, which is less than 40? For x=14, A+54 ≤ -14 +54=40, D=32<40, but for specific A, when A is smaller, A+54 is smaller.

For example, in x=14, A=-23, A+54=31, D=32>31, so ok.

A=-22, A+54=32, D=32, equal, so on the first day, which is included.

Similarly for others.

The smallest A+54 can be: since A ≥ -x-9, and x is given, for each x, we have A ≥ -x-9, so A+54 ≥ -x-9+54=45-x.

For x=14, 45-14=31, D=32>31, so ok.

For x=17, A≥-26? Earlier for x=17, A≥-26? But we have A≥-17 from the max, but actually the lower bound for A is max(-26,-17,-39)=-17, so A≥-17, A+54≥37, D=44>37, good.

But D=4x-24, and we need D ≥ A+54, but since A ≤ -x, so A+54 ≤ 54 - x.

So D ≥ 54 - x? Not necessarily, because A could be smaller, but in the constraint, for each A, it might be different.

But in our earlier explicit check, for each case, D is within [A+54, A+61], so no problem.

To confirm, for x=14, min A+54 is min over possible A, which is -23+54=31, and D=32≥31, and max A+54= -22+54=32, D=32≤32, so always D ≥ min A+54, and D ≤ max A+61.

But in any case, we already verified.

Now, so sum is 62.

But perhaps the answer is 62, but let me see why it might not be.

Another thing: in the egg stage, x days prior, and it was in egg stage, but if it was on the first day or last, but we included.

But perhaps for the butterfly stage, if it is on the last day, and the stage ends, but in our definition, it is included.

But in some interpretations, the last day might not be included, but typically it is.

Perhaps the life cycle stages are exclusive at endpoints, but that would be messy.

To be safe, let me check for x=17.

On day 44, butterfly: for A=-17, butterfly days 37 to 44 inclusive, so day 44 is the last day, so if we include it, it's fine.

Similarly, for egg on day -17, first day, included.

Now, for x=14, when A=-23, day -14 is last day of egg, included.

Day 32, for A=-23, butterfly from 31 to 38, day 32 is inside.

But for A=-22, day -14: egg days -22 to -13, so -14 is included, but is it the start or end? It's in the middle, no problem.

Day 32: for A=-22, first day of butterfly, which we included.

Now, if someone argues that on the first day of butterfly, it is not yet butterfly, but that would be unusual.

Similarly, for chrysalis transition.

But in standard interpretation, when we say the stage lasts d days, it includes d consecutive days, so the start and end days are included.

To confirm, total life from egg start to butterfly end: from A to A+61 inclusive, which is 62 days? (61-0)+1=62 days? But stages: egg 10, caterpillar 30, chrysalis 14, butterfly 8, total 10+30+14+8=62 days, and from day A to A+61 inclusive is 62 days, yes, so it matches, so inclusive is correct.

So should be fine.

Now, but still, sum is 62.

But perhaps I need to include more x.

Earlier for x=15, we had multiple A, but x is still 15.

But the sum is over x, not over scenarios.

Now, but in the answer choices, there is 63, which is odd, so maybe I missed a value.

What about x=10? But earlier x must be at least 14.

Perhaps when 4x-24 is less than the chrysalis start, but we have.

Another thing: the expression "4x - 24" might be interpreted as the day, but also, it must be that on that day it is butterfly, but we have.

But for the same x, different A might give different, but in our case, for each x, we have at least one A that works.

But for x=15, we have A from -24 to -18, all work, so x=15 is valid.

Similarly.

Now, but let me check x=13 again, but we saw max L = -22, min U = -26, -22 > -26, no.

x=18: max L = max(-27, -13, -39) = -13, min U = min(-18,-6,-10) = -18, -13 > -18, no.

x=19: max L = max(-28,4*19-85=76-85=-9,-39) = max(-28,-9,-39) = -9

min U = min(-19,4*19-78=76-78=-2,-10) = min(-19,-2,-10) = -19

-9 > -19, no.

x=12: as before, no.

x=20: max L = max(-29,80-85=-5,-39) = -5

min U = min(-20,80-78=2,-10) = min(-20,2,-10) = -20

-5 > -20, no.

Now, lower x, say x=10.

Egg: A≥-10-9=-19, A≤-10

Butterfly: $A \geq 40-85=-45$, $A \leq 40-78=-38$

Max L = max(-19,-45,-39) = -19? approx

Min U = min(-10,-38,-10) = -38? min(-10,-38,-10) = -38

$-19 \leq A \leq -38$? $-19 > -38$, no.

x=11: $A \geq -11-9=-20$, $A \leq -11$

But fly: $A \geq 44-85=-41$, $A \leq 44-78=-34$

Max L=max(-20,-41,-39) = -20

Min U=min(-11,-34,-10) = -34

-20 > -34? no, but -20 is greater than -34, so no.

x=13: as before.

Now, what about higher x? x=17 is max.

But for x=17, it worked.

Now, but in x=17, A=-17, and caterpillar constraint $A \leq -10$, which is satisfied.

But is there a constraint that the butterfly stage must be after day 0? But it is.

Perhaps for some x, 4x-24 is negative, but not in this range.

Another thing: in the expression "x days prior", x must be positive, which it is, since days prior.

In the problem, "in the egg stage x days prior to that day", so x should be positive, which it is for all our values.

Now, but sum is 62, but let me see if x=13.5, but x is integer, as per sum of integer values.

Perhaps I missed that for x=14, when A=-22, but something.

Another thought: in the caterpillar stage, it lasts 30 days, and on day 0 it is caterpillar.

The prior stage is egg, which ended before.

In the egg stage x days prior, which is day -x.

Since egg ends at day A+9, and caterpillar starts at day A+10, so the last day of egg is day A+9, first day of caterpillar is day A+10.

On day -x, it was in egg, so $-x \leq A+9$.

But also, since caterpillar starts at A+10, and on day 0 it is caterpillar, so $A+10 \leq 0$.

We have $A+10 \leq 0$ from caterpillar start.

Now, on day -x, in egg, so $A \leq -x \leq A+9$.

Now, from A ≤ -x and A+10 ≤ 0, so -x ≥ A, and A+10 ≤ 0, so -x ≥ A ≥ -10? From A+10 ≤ 0, A ≤ -10.

But also A ≥ -x -9.

But from A ≤ -x and A ≤ -10, and A ≥ -x -9.

In our earlier, we have A ≤ -10.

But for the egg on day -x, since egg ends at A+9, and -x ≤ A+9.

But from A ≤ -x, so -x ≥ A, and -x ≤ A+9, which is consistent.

Moreover, since caterpillar starts at A+10, and on day 0 it is caterpillar, so day 0 ≥ A+10.

Now, the butterfly stage: it must be that the butterfly stage starts after the caterpillar ends, but in our timeline it is.

Specifically, the earliest butterfly day is A+54, and since caterpillar ends at A+39, so A+54 > A+39, so always after.

But we need to ensure that on day D=4x-24, it is not during chrysalis or something, but we already checked that it is in butterfly.

In our explicit check, for each case, it is within butterfly days.

For example, in x=14, A=-23, butterfly 31-38, D=32, good.

But for A=-22, same x, butterfly 32-39, D=32, good.

Now, is there a case where it is not? For example, in x=15, D=36, and for different A, butterfly starts as early as 30 (for A=-24), up to 38 (A=-18? -18+54=36), A min is -24, A+54=30, A+61=37? Earlier for A=-24, butterfly 30 to 37.

Day D=36 for x=15, 36 ≤37? 37 is included, so yes.

For A=-18, butterfly from -18+54=36 to -18+61=43, so day 36 is first day, included.

All good.

Now, so sum is 62.

But perhaps the answer is 62, but let me see the options; maybe I need to consider that for some x, it works only for certain A, but x is still counted.

But in the sum, it's the values of x, so each x appears once.

Now, another thing: in x=17, it works, but let me see if x=16 or others.

Perhaps x=11, but we saw not.

Maybe when 4x-24 is exactly the start or end, but we included.

Perhaps the problem is that "in the butterfly stage" implies it is not in transition, but since we have discrete days, it should be fine.

Maybe the stages overlap or something, but no.

Another idea: perhaps for the same x, with different A, but the butterfly day is different, but in our case for fixed x, D is fixed, and we adjust A to make it work, which we did.

Now, but in the problem, A is fixed for a given butterfly, but for each possible x, we can choose A such that it satisfies, so each x is possible.

To confirm, for each x from 14 to 17, there exists A such that all conditions hold.

So the possible x are 14,15,16,17.

Sum is 14+15+16+17.

Now, 14+17=31, 15+16=31, 31+31=62.

14+15=29, 29+16=45, 45+17=62.

Now, perhaps the answer is 62.

But let me see why it might be 63; maybe they include x=13 or x=18, but they don't work.

Perhaps I miscalculated the range.

Another thought: in the caterpillar stage, when it is on day 0, and it lasts 30 days, so it could be early or late in the stage.

In our case, for different A, the position in caterpillar changes.

For example, for x=14, if A=-23, caterpillar from -13 to 16, day 0 is relatively early.

If A=-22, caterpillar from -12 to 17, day 0 is still early.

But for x=15, A=-24, caterpillar -14 to 15, day 0 is near the end? Day 15 is the last day, so day 0 is 15 days after start? Start at -14, so day -14 to 15, so day 0 is the 15th day? From -14 to 0 inclusive: days -14,-13,...,0, that's 15 days? From -14 to 0 inclusive: number is 0 - (-14) +1=15, but caterpillar lasts 30 days, so on day 0, it is the 15th day, so half way, fine.

For x=17, A=-17, caterpillar -7 to 22, so on day 0, it is the 8th day? From -7 to 0: days -7,-6,...,0, that's 8 days, so about quarter way, fine.

Now, no issue.

Perhaps the butterfly stage: for x=17, day 44, which is the last day, and if it dies the next day, but still on that day it is butterfly.

Now, so I think 62 is correct.

But let me check the sum again: 14+15+16+17.

14+17=31, 15+16=31, 62.

14+16=30, 15+17=32, 30+32=62.

Same.

Now, perhaps the answer is A)62.

But the problem says "the sum of the integer values that x can take", and I have four values, sum 62.

But in the options, B is 63, which might be if I included x=13 or something.

Perhaps for x=14, it doesn't work for both A, but we saw it does.

Another thing: in the expression "4x - 24", and for butterfly, but perhaps it must be that it is after the chrysalis, which it is.

I recall that the chrysalis stage is 14 days, but in our timeline, from caterpillar end to butterfly start is one day? From last caterpillar day A+39 to first chrysalis day A+40, so next day is chrysalis.

Then chrysalis from A+40 to A+53 inclusive, 14 days.

Butterfly starts at A+54.

The duration from end of caterpillar to start of butterfly is (A+54) - (A+39) = 15 days, but the chrysalis is only 14 days? Inconsistency.

Problem: caterpillar ends at day A+39.

Chrysalis starts at day A+40.

Chrysalis lasts 14 days: if from A+40 to A+53 inclusive, then the days are A+40,A+41,...,A+53. Number: 53-40+1=14 days.

Then butterfly starts at day A+54.

But from caterpillar end A+39 to butterfly start A+54 is 15 days later, but chrysalis is only 14 days? So what happens in between?

There is a gap? But in the life cycle, after caterpillar, it becomes chrysalis immediately, so probably no gap.

The issue is in the transition.

Typically, the end of one stage is the start of the next.

So, when caterpillar stage ends, it immediately becomes chrysalis.

So, on day A+39, it is the last day of caterpillar, but at the end of the day, it turns into chrysalis, so on the next day, it is already in chrysalis.

But the day A+39 is fully caterpillar, and day A+40 is fully chrysalis.

The chrysalis stage should include the time when it is in chrysalis, which starts at the beginning of day A+40.

But the duration: if it lasts 14 days, then it should end at the end of day A+53, so that the next day A+54 is butterfly.

> But the number of days from start to end: day A+40 to A+53 inclusive is 14 days: 40,41,42,43,44,45,46,47,48,49,50,51,52,53. That's 14 days.
>
> Similarly for others.
>
> But is there a day missing? From caterpillar end to chrysalis start: caterpillar ends at end of day A+39, chrysalis starts at start of day A+40, so no gap, but no overlapping day.
>
> The problem is that day A+39 is caterpillar, not chrysalis, so the chrysalis stage starts after.
>
> But in terms of the day when it is in chrysalis, it is days A+40 to A+53.
>
> Similarly, no issue.
>
> But now, the butterfly stage: for it to be in butterfly on a certain day, we defined days A+54 to A+61.
>
> Now, in our earlier calculations, for the days we care about, it was within, so should be fine.
>
> To confirm, total days: egg: A to A+9 (10 days), caterpillar: A+10 to A+39 (30 days? From A+10 to A+39 inclusive: (39-10)+1=30, yes), chrysalis: A+40 to A+53 (14 days: 53-40+1=14), butterfly: A+54 to A+61 (8 days: 61-54+1=8), total 10+30+14+8=62, and consecutive days from A to A+61 inclusive, 62 days, good.
>
> So no gap, no overlap.
>
> Thus, our setup is correct.
>
> Therefore, the sum is 14+15+16+17=62.
>
> So the answer should be 62.

The life cycle stages are defined as follows, with the start of the egg stage at day A:

Egg stage: days A to $A+9$ inclusive (10 days).

Caterpillar stage: days $A+10$ to $A+39$ inclusive (30 days).

Chrysalis stage: days $A+40$ to $A+53$ inclusive (14 days).

Butterfly stage: days $A+54$ to $A+61$ inclusive (8 days).

Given that the butterfly is in the caterpillar stage on day 0 (1st of April), the constraints are:

$$A+10 \leq 0 \leq A+39$$

Solving for A:

with A being an integer.

The butterfly was in the egg stage x days prior to day 0, so on day $-x$. This requires:

$$A \leq -x \leq A+9$$

which simplifies to:

$$-x - 9 \le A \le -x$$

The butterfly will be in the butterfly stage $4x - 24$ days after day 0, so on day $4x - 24$. This requires:

$$A + 54 \le 4x - 24 \le A + 61$$

Solving for A:

$$4x - 85 \le A \le 4x - 78$$

Combining all constraints, A must satisfy:

$$\max(-x - 9, 4x - 85, -39) \le A \le \min(-x, 4x - 78, -10)$$

with A integer. Additionally, the inequalities must be consistent, so:

$$\max(-x - 9, 4x - 85, -39) \le \min(-x, 4x - 78, -10)$$

Necessary conditions derived from pairwise inequalities are:

From $A \le -x$ and $A \ge 4x - 85$: $x \le 17$.

From $A \ge -x - 9$ and $A \le 4x - 78$: $x \ge 13.8$, so $x \ge 14$ since x is integer.

Thus, x ranges from 14 to 17 inclusive. Checking each integer value:

For $x = 14$:

$\max(-23, -29, -39) = -23$, $\min(-14, -22, -10) = -22$, so $-23 \le A \le -22$.

$A = -23$ or $A = -22$ satisfies all constraints, and the stages are consistent.

For $x = 15$:

$\max(-24, -25, -39) = -24$, $\min(-15, -18, -10) = -18$, so $-24 \le A \le -18$.

A from -24 to -18 inclusive satisfies all constraints, and the stages are consistent.

For $x = 16$:

$\max(-25, -21, -39) = -21$, $\min(-16, -14, -10) = -16$, so $-21 \le A \le -16$.

A from -21 to -16 inclusive satisfies all constraints, and the stages are consistent.

For $x = 17$:

$\max(-26, -17, -39) = -17$, $\min(-17, -10, -10) = -17$, so $A = -17$.

$A = -17$ satisfies all constraints, and the stages are consistent.

Values of x outside 14 to 17 do not satisfy the constraints. For example:

$x = 13$: $\max(-22, -33, -39) = -22$, $\min(-13, -26, -10) = -26$, and $-22 > -26$, no solution.

$x = 18$: $\max(-27, -13, -39) = -13$, $\min(-18, -6, -10) = -18$, and $-13 > -18$, no solution.

The integer values of x are 14, 15, 16, and 17. The sum is:

$$14 + 15 + 16 + 17 = 62$$

\boxed{62}

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